# 2 Reference Systems

Reference systems are introduced in order to model geodetic observations as a function of unknown parameters of interest. The coordinate systems are defined in terms of orientation, metrics, and curvature; they are three-dimensional in principle (HEITZ 1988). A fourth dimension, time, enters through the mutual motion of the earth and other celestial bodies and through the earth's deformations. As with the earth, reference systems can be defined for the moon and the planets in the solar system.

Basic units and fundamental constants are foundational to the geodetic measurement and modeling process [2.1]. Time systems are based either on processes of quantum physics or on the daily rotation of the earth [2.2 ]. Global reference systems are realized through reference frames, e.g., the ITRF, which is established and maintained by the International Earth Rotation Service [2.3]. We distinguish between the space-fixed celestial reference system [2.4] and the earth-fixed terrestrial reference system [2.5] (KOVALEVSKY et al. 1989). In addition, gravity related reference systems have to be introduced, as most geodetic observations refer to the earth's gravity field [2.6].

## 2.1 Basic Units and Fundamental Constants

Length, mass, and time are basic quantities used in geodesy. The units for these quantities are the meter (m), the kilogram (kg), and the second (s) respectively. They are defined through the International System of Units (Système International d'Unités SI), established in 1960 by the 11th General Conference of Weights and Measures (CGPM) in Paris (MARKOWITZ 1973, BIPM 1991). The definitions are as follows:

- The *meter* is the length of the path traveled by light in vacuum during a time interval of 1/299 792 458 of a second (CGPM 1983).
- The *kilogram* is the unit of mass; it is equal to the mass of the international prototype of the kilogram (CGPM 1901).
- The *second* is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom (CGPM 1967).

The establishment and maintenance of the reference standards for these units is the task of the *Bureau International des Poids et Mésures* (BIPM), located in Sèvres, France. BIPM cooperates with the national laboratories of standards under the guidelines of the International Meter Convention (1875). These national laboratories include the National Institute of Standards and

Technology, Gaithersburg, Md., U.S.A.; the National Physical Laboratory, Teddington, U.K.; and the Physikalisch-Technische Bundesanstalt, Braunschweig, Germany.

The *realization* of the *meter* is based on interferometric measurements (relative uncertainty $10^{-12}$) using light with highly stable frequencies (stabilized lasers). The international *kilogram* prototype has been kept in BIPM since 1889; national prototypes are related to it with an uncertainty of $10^{-9}$. The BIPM Time Section (until 1987: Bureau International de l'Heure BIH, Paris) defines the *second* (relative uncertainty $10^{-14}$) and the atomic time scale, cf. [2.2.1].

*Previous definitions* of the meter and the second were based on natural measures. The *meter* was intended to be one ten-millionth part of the meridian quadrant passing through Paris. Its length was derived from an arc measurement, cf. [1.3.2], and realized in 1799 by a prototype meter bar called "mètre des archives" (legal meter). Following the International Meter Convention, a more stable version (platinium-iridium bar) was manufactured (international meter). It has been preserved since 1889 at the BIPM. This definition (uncertainty $10^{-7}$) was valid until 1960 when, for the first time, the wavelength of a certain spectral line of light became the defining quantity.

Since ancient times, the natural measure for *time* has been the daily rotation of the earth about its axis. The mean solar day, cf. [2.2.2], was determined by astronomic observations, and the second was defined as 1/86400 part of that day. From the 1930's on, it became obvious that this definition was uncertain by about $10^{-7}$ due to the irregularities of the earth's rotation, cf. [2.5.2].

As a supplementary SI unit, the *radian* (rad) is used for *plane angles* :

- The radian is the plane angle between two radii of a circle subtended by an arc on the circumference having a length equal to the radius.

Geodesy, astronomy, and geography also use the *sexagesimal graduation* with 1 full circle = 360° (degrees), 1° = 60′ (minutes), and 1′ = 60″ (seconds, also arcsec). With $2\pi$ rad corresponding to 360°, an angle $\alpha$ is transformed from radian to degree by

$$\alpha(°) = \rho(°)\alpha\,\text{rad},\ \rho° = 180°/\pi\,. \tag{2.1}$$

Among the *fundamental constants* used in geodetic models is the *velocity of light* in a vacuum, which is by definition (1983)

$$c = 299\,792\,458\,\text{ms}^{-1}\,, \tag{2.2}$$

and the *gravitational constant* (CODATA system of physical constants 1986), which is defined as

$$G = (6.672\,59 \pm 0.000\,85) \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}\,. \tag{2.3}$$

Cavendish carried out the first experimental determination of G in 1798 with a torsion balance. Current work concentrates on increasing the relative accuracy of $G$ to better than $10^{-4}$. This includes investigations into dependence of $G$ on material, external influences, distance and direction, as well as non-inverse-square properties of gravitation (GILLIES 1987, FISCHBACH and TALMADGE 1999).

Other units and constants used in geodesy, astronomy, and geophysics will be introduced in the corresponding chapters, see also AHRENS (1995), BURŠA (1995), GROTEN (2000).

## 2.2 Time Systems

Time plays a fundamental role in geodesy. This is due to the fact that most measurement methods use the signal travel-time of electromagnetic waves for positioning, and that a uniform time scale is also needed in order to model the motion of artificial satellites. On the other hand, a time system is required for describing the relative motion of the earth in the solar system with respect to inertial space and for describing earth deformations due to internal and external forces.

Time systems are defined by the unit for a time interval and by a time epoch. They are based either on the definition of the SI second [2.2.1] or on the diurnal rotation of the earth about its axis [2.2.2]. Fundamental descriptions of time systems are found in MUELLER (1969), MORITZ and MUELLER (1987), SEIDELMANN (1992).

### 2.2.1 Atomic Time, Dynamical Time

A uniform time-scale of high accuracy is provided by the *International Atomic Time* (Temps Atomique International: TAI). It corresponds to the definition of the SI second, cf. [2.1], which has been made approximately equal to the second of the formerly used ephemeris-time. The latter was defined by the motion of the earth about the sun and determined through long-term astronomic observations. The origin of TAI was chosen so that its epoch (January 1, 1958, 0 h) coincided with the corresponding epoch of Universal Time UT1, cf. [2.2.2]. The TAI day comprises 86400 s, and the Julian Century has 36525 TAI days.

TAI is *realized* by a large set (more than 200) of atomic clocks (mostly cesium beam frequency standards providing long-term stability and a few hydrogen masers) maintained at about 60 laboratories around the world. Clock comparisons are performed at a number of timing centers, employing GPS observations for time links, cf. [5.2.5]. From these local determinations, a weighted mean is calculated at the BIPM Time Section. The relative frequency-stability of TAI is between a few $10^{-15}$ (over minutes to days) and $10^{-13}$ (over years). Due to relativistic effects, the readings of the atomic clocks are reduced to a common height reference (SI second "on the geoid").

The motions of celestial bodies and artificial satellites have to be described by a strictly uniform time scale (inertial time). This is provided by a *dynamical time*, which is based on motions of bodies in the solar system. Dynamical time scales refer either to the barycenter of the solar system (Barycentric Dynamic Time TDB) or to the geocenter (Terrestrial Time TT). The TT unit is practically equivalent to TAI, with a constant difference resulting from the epoch definition of TAI:

$$TT = TAI + 32.184\,s\,. \tag{2.4}$$

Dynamical time is used in celestial mechanics with Newton's equations of motion, e.g., as an argument for the astronomical ephemeris of the moon and the sun.

### 2.2.2 Sidereal and Universal Time

The diurnal rotation of the earth provides a natural measure for time. Corresponding time systems are introduced in order to relate earth-based observations to a space-fixed system: Sidereal and Universal (solar) Time. Hereby, two periodic motions of the earth play a role (Fig. 2.1):

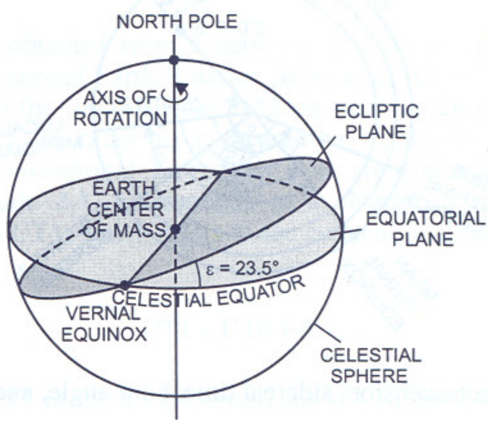


*Fig. 2.1* Earth rotation, equatorial plane, and ecliptic plane

- the *diurnal rotation* (spin) of the earth about its polar axis. This rotational axis approximately coincides with the axis of maximum moment of inertia, and it passes through the earth's center of mass, cf. [2.5.2]. The *equatorial plane* is perpendicular to the axis of rotation,
- the *annual revolution* of the earth around the sun. Following Kepler's laws, the earth describes an ellipse with the sun at one of its focal points. Minor perturbations arise due to the gravitation of the moon and other planets. The plane of the earth's orbit is called the *ecliptic plane*; it has an obliquity of about 23.5° with respect to the equatorial plane.

By circumscribing the unit sphere around the center of the earth, simple geometric relations are obtained. The *celestial equator* and the *ecliptic* are defined by the intersections of the sphere with the corresponding planes. The *vernal equinox* (also first point of Aries) is the intersection of the ecliptic and the equator where the sun passes from the southern to the northern hemisphere.

*Sidereal time* is directly related to the rotation of the earth. *Local Apparent* (or true) *Sidereal Time* (LAST) refers to the observer's (local) meridian; it is equal to the hour angle of the (true) vernal equinox (Fig. 2.2), cf. [2.4.1]. The vernal equinox is affected by precession and nutation and experiences long and short-periodic variations, cf. [2.4.2]. If nutation is taken into account, we obtain *Local Mean Sidereal Time* (LMST), referring to the mean vernal equinox. For the Greenwich meridian the corresponding hour angles are called *Greenwich Apparent Sidereal Time* (GAST) and *Greenwich Mean Sidereal Time* (GMST). The astronomic longitude $\Lambda$ is the angle between the meridian planes of the observer and Greenwich. It is given by, cf. [2.6.2]

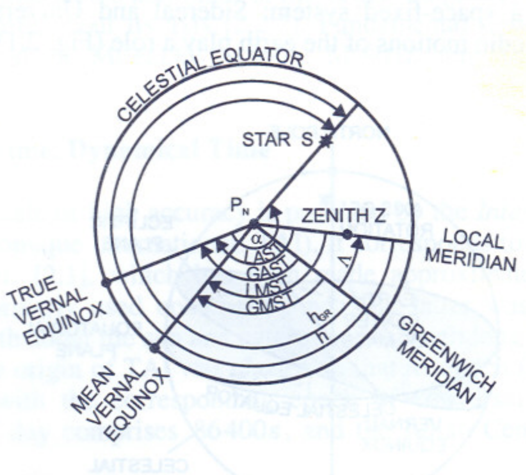


*Fig. 2.2.* Rectascension, sidereal time, hour angle, and longitude

$$\Lambda = \text{LAST} - \text{GAST} = \text{LMST} - \text{GMST}. \qquad (2.5)$$

LAST is determined from astronomical observations to fixed stars and extragalactic radio sources. The mean sidereal time scale is still affected by precession (long-periodic). The *mean sidereal day* is the fundamental unit; it corresponds to the time interval of two consecutive transits of the mean vernal equinox through the meridian.

For practical reasons, *solar time* is used in everyday life. It is related to the apparent diurnal motion of the sun about the earth. Since this revolution is not uniform, a "mean" sun is introduced which moves with constant velocity along the equator and coincides with the true sun at the vernal equinox. *Mean solar time* is equal to the hour angle of the mean sun plus 12 hours. If referred to the Greenwich mean astronomical meridian, cf. [2.5.1], it is termed *Universal Time* (UT). Its fundamental unit is the *mean solar day,* being the interval between two transits of the fictitious sun through the meridian.

The conversion of Universal Time to Greenwich Mean Sidereal Time is rigorously possible and is given by a series development with time defined by the International Astronomical Union (MORITZ and MUELLER 1987). Since the orbital motion of the earth is about 1° per day $(360°/365\,\text{d})$, the year has one day more in sidereal days than in solar days. We have the following approximation:

$$1 \text{ mean sidereal day} = 1 \text{ mean solar day} - 3\text{m}\,55.90\text{s} = 86164.10\text{s}. \qquad (2.6)$$

The earth's rotation rate is $15.041\,07''/1\,\text{s}$, and its angular velocity is

$$\omega = 2\pi/86\,164.10\,\text{s} = 7.292\,115 \times 10^{-5}\,\text{rad}\,\text{s}^{-1}. \qquad (2.7)$$

Universal time is obtained from a network of stations operating within the frame of the International Earth Rotation Service, cf. [2.3]. The observed local time UT0 refers to the instantaneous rotation axis, which is affected by polar motion, cf. [2.5.2]. In order to compare the results of different stations, reductions to a *Conventional Terrestrial Pole* are applied. The reduction in astronomic longitude $\Delta\Lambda_P$ corresponds to a change in time, cf. [5.3.3]. It transforms UT0 to UT1, which refers to the conventional terrestrial system, cf. [2.5.3]:

$$\text{UT1} = \text{UT0} + \Delta\Lambda_P. \qquad (2.8)$$

The precision of UT1 is about 0.01 to 0.02 ms at a 1d resolution.

UT1, as well as Greenwich Mean Sidereal Time, still contains the variations of the earth's rotation with time, which are of secular, periodic, and irregular character, cf. [2.5.2]. An approximation to a uniform time scale can be achieved by modeling the seasonal variations of annual and semiannual type. With the corresponding reduction $\Delta\Lambda_S$, we obtain

$$UT2 = UT0 + \Delta\Lambda_P + \Delta\Lambda_S \,. \tag{2.9}$$

A practical time scale, as needed in navigation for instance, has to provide a uniform unit of time and maintain a close relationship with UT1. This led to the introduction of the *Coordinated Universal Time* (UTC). Its time interval corresponds to atomic time TAI, cf. [2.2.1], and its epoch differs by not more than 0.9s from UT1. In order to keep the difference

$$|DUT1| = |UT1 - UTC| < 0.9\,s\,, \tag{2.10}$$

"leap seconds" are introduced to UTC when necessary. UTC is provided by the BIPM Time Section and broadcasted by time signal stations, while DUT1 is calculated by the IERS, cf. [2.3].

Among the continuously broadcasting time stations are DCF77/Mainflingen (77.5 kHz), HBG/Prangins (75 kHz); MSF/Rugby (60 kHz) in Europe; WWV resp. WWVB/Ft. Collins, Colorado (2500 to 20000 kHz resp. 60 kHz); and WWVH/Kauai, Hawaii (2500 to 15000 kHz).

## 2.3 International Earth Rotation Service

The International Earth Rotation Service (IERS) is in charge of providing and maintaining conventional celestial and terrestrial reference frames. These frames are a realization of the reference systems recommended by the International Astronomical Union (IAU) and the International Union of Geodesy and Geophysics (IUGG). IERS is also responsible for the determination of the orientation parameters of the earth as functions of time, which relate the two frames to each other (SEIDELMANN 1992, REIGBER and FEISSEL 1997).

Established by the IAU and IUGG, the IERS has operated since January 1, 1988. It collects, analyzes, and models observations of a global network of astronomic and geodetic stations (about 300 sites in 1996), operating either permanently or for a certain time span. Observation techniques include Very Long Baseline Interferometry (VLBI), Lunar Laser Ranging (LLR), Global Positioning System (GPS), Satellite Laser Ranging (SLR), and DORIS (Doppler Orbit determination and Radio positioning Integrated on Satellite), cf. [5.2] to [5.3].

The different types of observations are evaluated at the respective IERS coordinating centers and then combined by an adjustment at the IERS Central Bureau. The results include the positions (coordinates) of both the extragalactic radio sources and the terrestrial stations, the earth orientation parameters (EOP), and other information. With respect to the EOP, VLBI provides information about precession, nutation, polar motion, and UT1, cf. [2.4.2], [2.2.2]. Satellite techniques contribute to the daily interpolation of UT and to the determination of polar motion, cf. [2.5.2]. The results are disseminated through bulletins, annual reports, and technical notes. The combined solutions have an accuracy of ±0.0003″ for EOP and ±0.01 m for the positions of the terrestrial stations, cf. [2.4], [2.5]. The evaluation of the observations is based on the IERS Conventions, which are consistent with the IAU and IUGG/IAG recommendations for reference systems (McCARTHY 1996), cf. [2.4.2], [4.3].

Among the early international agreements on positioning and time were the introduction of the Greenwich zero meridian and Universal Time (1884). The first fundamental catalogue of selected stars was published in the 1880's, which started a series of star catalogues providing the positions of fixed stars.

International activities of monitoring the earth's rotation date back to 1899, when the *International Latitude Service* (ILS) started to determine polar motion through latitude observations at five observatories located around the globe on the 39°08′ northern parallel. After extension to the *International Polar Motion Service* (IPMS), and in cooperation with the *Bureau International de l'Heure* (BIH) established in 1912, about 50 astronomical observatories contributed to the determination of polar motion and time. An accuracy of ±0.02″ resp. ±1 ms was reached for mean values over 5 days. IPMS and the earth rotation section of BIH have been replaced by IERS, while the BIH activities on time are continued at the BIPM, cf. [2.2.1].

## 2.4 Celestial Reference System

An inertial system is needed in order to describe the motions of the earth and other celestial bodies in space, including those of artificial satellites. Such a system is characterized by Newton's laws of motion; it is either at rest or in the state of a uniform rectilinear motion without rotation. A space-fixed system (celestial reference system) represents an approximation to an inertial system and can be defined by appropriate conventions: Conventional Inertial System (CIS). The coordinate frame for such a system is provided by spherical astronomy [2.4.1]. The spatial orientation of this frame varies with time, and therefore, modeling of the variations is required [2.4.2]. The International Celestial Reference Frame represents the realization of the celestial reference system [2.4.3], KOVALEVSKY et al. (1989), SEIDELMANN (1992).

### 2.4.1 Equatorial System of Spherical Astronomy

The coordinates of the celestial reference-system are defined by the *equatorial system* of spherical astronomy (MUELLER 1969, EICHHORN 1974). We introduce a three-dimensional Cartesian coordinate system with the origin at the center of mass of the earth (geocenter). The *Z*-axis coincides with the rotational axis of the earth. The *X* and *Y*-axes span the equatorial plane, with the *X*-axis pointing to the vernal equinox and the *Y*-axis forming a right-handed system (Fig. 2.3), cf. [2.2.2].

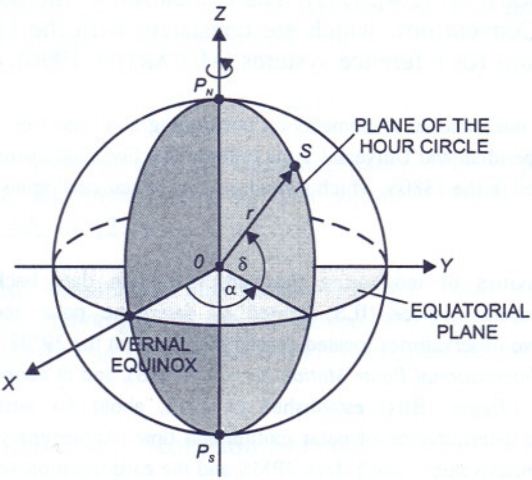


*Fig. 2.3.* Astronomic equatorial system

In the sequel, we shall also shift the origin of this system to the position of an observer on the earth (topocenter) or to the barycenter of the solar system. The directions to celestial bodies then vary with different definitions of the origin (parallaxes), cf. [5.3.3]. Since the earth's radius is negligibly small compared to the distances to stars and extragalactic radio sources, no distinction is necessary between a topocentric and a geocentric system.

We circumscribe the unit sphere (celestial sphere) about the earth. The rotational axis meets the sphere at the celestial north and south poles $P_N$ and $P_S$. The great circles perpendicular to the celestial equator, which contain the celestial poles, are called *hour circles*, and the small circles parallel to the equator are termed *celestial parallels*.

The *right ascension* $\alpha$ is the angle measured in the plane of the equator between the planes of the hour circles passing through the vernal equinox and the celestial body *S*; it is reckoned from the vernal equinox anticlockwise. The *declination* $\delta$ is the angle measured in the plane of the hour circle between the equatorial plane and the line $OS$ (positive from the equator to $P_N$ and negative to $P_S$).

The *position* of a *celestial body* S can be described either by the Cartesian coordinates $X,Y,Z$, or by the spherical coordinates $\alpha,\delta,r$ ($r$ = distance from the origin $O$). We have the transformation

$$\mathbf{r} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = r \begin{pmatrix} \cos\alpha\cos\delta \\ \sin\alpha\cos\delta \\ \sin\delta \end{pmatrix}. \tag{2.11}$$

In geodesy, only directions are important for stars and extragalactic sources. With $r = 1$, $\alpha$ and $\delta$ describe the position of $S$ on the unit sphere. They can also be expressed by the lengths of the corresponding arcs on the equator and the hour circle.

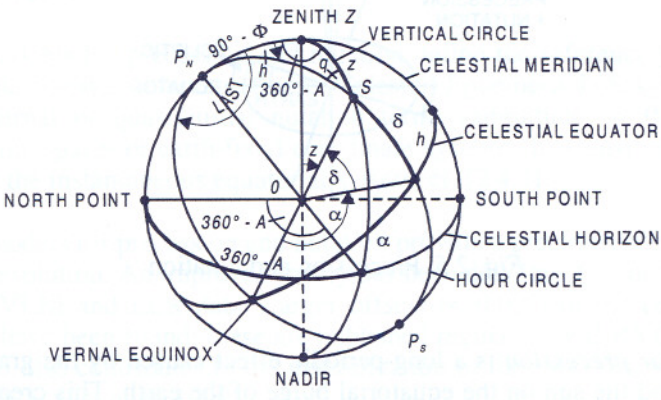


*Fig. 2.4.* Astronomic equatorial and horizon system

We introduce the local *meridian plane* of the observer, spanned by the local vertical (direction of the plumb line) and the rotational axis, after a parallel shift from the geocenter to the topocenter. The *zenithal point* $Z$ is the intersection of the vertical with the unit sphere, and the *celestial meridian* is the great circle through $Z$ and the poles (Fig. 2.4). The *hour angle* $h$ is measured in the equatorial plane between the celestial meridian through $Z$ and the hour circle of $S$, reckoned from the upper meridian toward west. Because of the earth's rotation, the hour angle system $(h,\delta)$ depends on time. The $h,\delta$-system is rotated, with respect to the $\alpha,\delta$-system, about the polar axis by the angle of sidereal time LAST, cf. [2.2.2]. We have the relation (Fig. 2.2)

$$\text{LAST} = h + \alpha\,, \tag{2.12}$$

which is used with time determination, cf. [5.3.2].

### 2.4.2 Precession and Nutation

The earth's axis of rotation, which has been introduced as the $Z$-axis, changes its spatial orientation with time. As a consequence, the position ($\alpha,\delta$) of a celestial body varies, with a superposition of long and short-periodic effects (MORITZ and MUELLER 1987, SEIDELMANN 1992, DICKEY 1995).

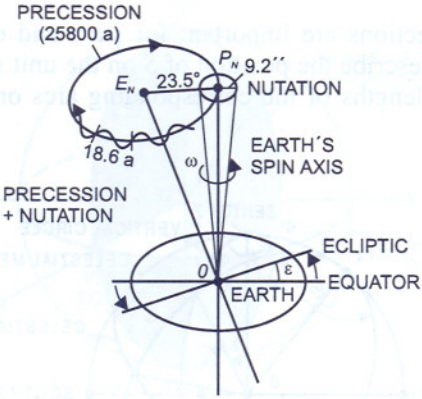


*Fig. 2.5.* Precession and nutation

The *lunisolar precession* is a long-periodic effect caused by the gravitation of the moon and the sun on the equatorial bulge of the earth. This creates a force couple (torque) which tends to turn the equatorial plane into the plane of the ecliptic (Fig. 2.5). In combination with the moment of the earth's rotation, the earth's axis describes a gyration of a cone with a generating angle of 23.5° (corresponding to the obliquity of the ecliptic $\varepsilon$), about the northern pole of the ecliptic $E_N$. The vernal equinox moves clockwise along the ecliptic at a rate of 50.3″/year, making a complete revolution in about 25800 years. The gravitation of the planets causes a slow dislocation of the earth's orbit and thereby an additional migration of the vernal equinox along the equator and a change in $\varepsilon$: *planetary precession*. The sum of the lunisolar and the planetary precession is termed *general precession*.

The precession is superimposed by short-periodic effects known as *nutation*, which has periods between 5 days and 18.6 years. These periods are mainly due to the time variations of the inclination of the moon's orbit with respect to the ecliptic (appr. 5°). Other components have semiannual and semimonthly periods and stem from the oscillations of the sun and moon between the earth's northern and southern hemisphere.

Precession and nutation can be modeled as a function of time using the ephemerides of the moon, the sun, and the planets. The IAU (1976) theory of *precession* provides three time-dependent Eulerian rotation angles for reducing the positions of celestial bodies to a common reference. For the reference epoch J 2000.0 (Julian epoch January 1, 2000, $12^h$ TDB, cf. [2.2.1]), we have the fundamental constants "general precession in longitude at the ecliptic" (5029.0965″/century) and "obliquity of the ecliptic" (23°26′21.412″).

The IAU (1980) theory of *nutation* describes this effect by a rotation about the cone of precession. The deviation of the true pole from the mean pole is modeled by two time-dependent parameters. Hereby, the earth is regarded as an elliptical, rotating, elastic, and ocean-free body with solid inner and liquid outer cores (WAHR 1981, SEIDELMANN 1992). For the epoch J 2000.0 the constant of nutation is 9.2025".

The IAU models for precession and nutation define the reference pole for the international celestial reference frame (Celestial Ephemeris Pole CEP). CEP is free of diurnal or quasidiurnal nutation terms (amplitudes < 0.001″) with respect to the space- or earth-fixed coordinate systems. It is also referred to as the pole of the instantaneous equatorial system, cf. [2.4.3].

The IAU models for precession and nutation provide a precision of ±0.001″ at 5 to 7 days resolution. An improved theory has been developed at the IERS based on recent VLBI and LLR data. Larger offsets (< 0.02″) of the celestial pole from CEP have been found; these are published regularly by IERS (MCCARTHY 1996). GPS also contributes to the determination of the short-periodic nutation terms (ROTHACHER et al. 1999).

According to an IAU recommendation, the IAU (1976/1980) models for precession and nutation shall be replaced by January 1, 2003. The new model is the IAU 2000A (precision ±0.0002″), as published in the IERS conventions 2000. CEP will then be substituted by the Celestial Intermediate Pole (CIP), as defined by the model for periods greater than two days together with additional time-dependent corrections provided by IERS.

The instantaneous position of a celestial body, cf. [2.4.1], is called *true position* at the epoch *t*. By accounting for nutation, we obtain the *mean position* at epoch *t*, which refers to the mean celestial equator and the mean vernal equinox, cf. [2.2.2]. If precession is also taken into account, we get the mean position at the reference epoch J 2000.0.

### 2.4.3 International Celestial Reference Frame

The International Celestial Reference System (ICRS), as recommended by IAU, is based on the general theory of relativity, with the time coordinate defined by the international atomic time (ARIAS et al. 1995, MA and FEISSEL 1997). ICRS approximates a space-fixed conventional inertial-system (CIS) with the origin at the barycenter of the solar system. It is assumed that no global rotation of the system exists. This implies that the defining sources are either free from proper motion (component of spatial motion tangent to the celestial sphere) or that this motion can be modeled. The coordinate axes are defined by the celestial reference pole and the vernal equinox as provided by the IAU models for precession and nutation. They are realized through mean directions to extraterrestrial fiducial objects: stellar or radio source CIS (MUELLER 1988), cf. [2.4.2].

The *stellar system* is based on the stars of the Fundamental Catalogue FK5 (FRICKE et al. 1988). It provides the mean positions ($\alpha,\delta$) and the proper motions (generally < 1″/year) of 1535 fundamental stars for the epoch J 2000.0, with precisions of ±0.01...0.03″ and ±0.05″/century respectively. A supplement to FK5 contains additional stars up to an apparent magnitude of 9.5. The mean equator and the mean vernal equinox for J 2000.0 are realized by the FK5 catalogue, with an accuracy of ±0.05″. Due to refraction uncertainties, earth-based astrometry can hardly improve this accuracy.

*Astronomic space missions* have significantly improved the realization of a stellar CIS. The HIPPARCOS astrometry satellite (ESA, 1989 – 1993) was used to construct a network by measuring large angles between about 100000 stars (up to an apparent magnitude of 9) covering the entire sky. The reference frame thus established provides an accuracy of ±0.001″ and ±0.0005″/year for proper motion (HIPPARCOS 1995, KOVALEVSKY et al. 1997). From improved FK5 data and HIPPARCOS results, an FK6 catalogue has been developed for a small number of stars (340 "astrometrically excellent"), resulting in an improvement of proper motion as compared to the HIPPARCOS catalogue (WIELEN et al. 1999). Future astrometric space missions will employ optical interferometry and thus increase the positional accuracy to ±0.00001″ (BROSCHE and DICK 1996).

The *radio source system* is based on extragalactic radio sources (quasars and other compact sources). It was adopted as ICRS by the IAU in 1997 and has superseded the previous stellar system (FK5) since 1998. Due to the large distances (> 1.5 billion light years), these sources do not show a measurable proper motion. The system is realized through the International Celestial Reference Frame (ICRF), established and maintained by IERS (MCCARTHY 1996, MA et al. 1998). ICRF contains the coordinates (equatorial system, epoch J 2000.0) of more than 600 objects. About 200 of them are well observed

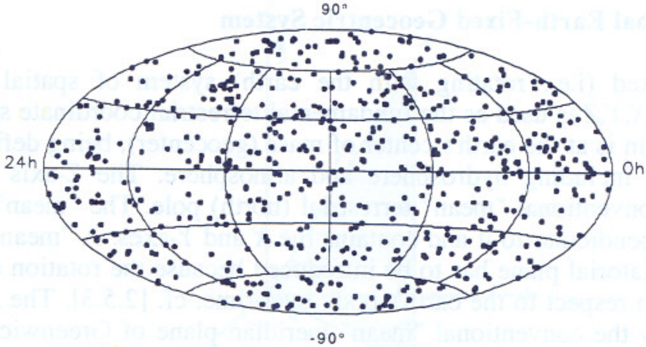


*Fig. 2.6.* International Celestial Reference Frame (ITRF), from IERS (1995): Missions and goals for 2000

"defining sources," and 100 more are used for densification and connection to the stellar-fixed reference system (Fig. 2.6). The southern sky is not as well covered, as the telescopes are concentrated in the northern hemisphere. The coordinates of the radio sources are determined by radio astronomy, with a precision of better than ±0.001″ on the average and ±0.0003″ for the most precisely observed objects (MA and FEISSEL 1997, BROSCHE und SCHUH 1999).

The link between the stellar and the radio source CIS is given with an accuracy of ±0.05...0.1″, which corresponds to the uncertainty of FK5. This connection will be improved by the results of the astrometric space missions (optical signals from a limited number of radio sources) to ±0.001″ or better for the epoch of observation.

## 2.5 Terrestrial Reference System

An *earth-fixed* reference system is introduced for positioning and navigation on and close to the earth's surface and for describing the earth's gravity field as well as other physical parameters. It is defined by a three-dimensional geocentric coordinate system [2.5.1]. The orientation of this system changes with time and with respect to the solid earth's body as well as to the celestial reference system [2.5.2]. The system is realized by the IERS International Terrestrial Reference Frame, which includes its relation to the International Celestial Reference Frame [2.5.3].

### 2.5.1 Global Earth-Fixed Geocentric System

An earth-fixed (i.e., rotating with the earth) system of spatial Cartesian-coordinates *X,Y,Z* is used as the fundamental terrestrial coordinate system (Fig. 2.7). Its origin is at the earth's center of mass (geocenter), being defined for the whole earth including hydrosphere and atmosphere. The *Z*-axis is directed towards a conventional "mean" terrestrial (north) pole. The "mean" equatorial plane is perpendicular to it and contains the *X* and *Y*-axes. A "mean" rotational axis and equatorial plane has to be introduced because the rotation of the earth changes with respect to the earth's body over time, cf. [2.5.3]. The *XZ*-plane is generated by the conventional "mean" meridian plane of Greenwich, which is spanned by the mean axis of rotation and the Greenwich zero meridian, to which Universal Time refers, cf. [2.2.2]. The *Z* and *X*-axes are realized indirectly through the coordinates of terrestrial "fiducial" stations, cf. [2.5.3]. The *Y*-axis is directed so as to obtain a right-handed system.

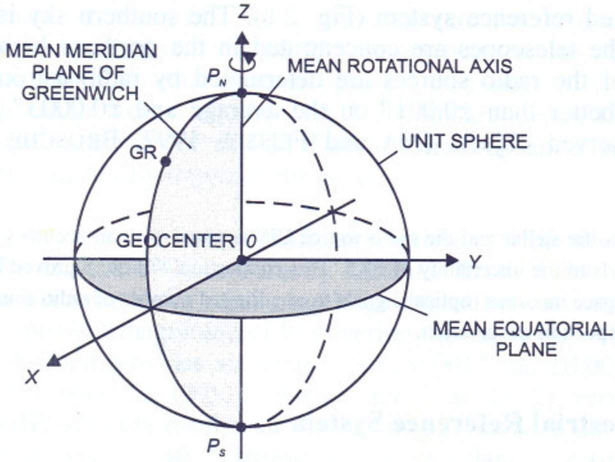


*Fig. 2.7.* Earth-fixed geocentric Cartesian system

The instantaneous axis of rotation is the common starting point for defining the *Z*-axes of the space-fixed and the earth-fixed reference systems. By referring to a reference epoch, the dependence on time is modeled, cf.[2.4.2], [2.5.2]. The directions of the *X*-axes of both systems differ by the angle of Greenwich mean sidereal time GMST, cf. [2.2.2].

In order to describe analytically certain physical properties of the earth (gravity field, magnetic field, topography, etc.), *spherical coordinates* $r, \vartheta, \lambda$ are employed. Here $r$ = radial distance from the geocenter, $\vartheta$ = polar distance (colatitude), and $\lambda$ = geocentric longitude. Instead of $\vartheta$, the geocentric latitude

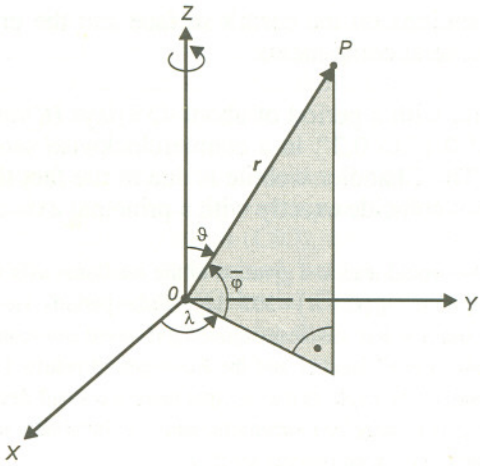


*Fig. 2.8.* Cartesian and spherical coordinates

$$\bar{\varphi} = 90^\circ - \vartheta \tag{2.13}$$

can be used (Fig. 2.8). The position of the point $P$ is then given by the positional vector

$$\mathbf{r} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = r \begin{pmatrix} \sin\vartheta\cos\lambda \\ \sin\vartheta\sin\lambda \\ \cos\vartheta \end{pmatrix}. \tag{2.14}$$

### 2.5.2 Polar Motion, Length of Day, Geocenter Variations

The rotation of the earth can be described by a vector directed to the north pole of the instantaneous axis of rotation and by the angular velocity $\omega$, see (2.7).

Direction and magnitude of the rotational vector *change with time* due to astronomical and geophysical processes. These processes include variations of the lunar and solar gravitation and mass redistributions in the atmosphere, the hydrosphere, the solid earth, and the liquid core. The changes are secular, periodic or quasiperiodic, and irregular in nature (LAMBECK 1980, MORITZ and MUELLER 1987, DICKEY 1995), cf. [8.3.1].

*Polar motion* (or wobble) is the motion of the rotation axis relative to the earth's crust as viewed from the earth-fixed reference system. It directly affects the coordinates of stations on the earth's surface and the gravity vector. Polar motion consists of several components:

- A free oscillation with a period of about 435 days (*Chandler period*), with an amplitude of 0.1″ to 0.2″, in a counterclockwise sense as viewed from the north pole. The Chandler wobble is due to the fact that the spin axis of the earth does not coincide exactly with a principal axis of inertia.

 For a rigid earth, this would lead to a gyration of the rotational axis about the principal axis of inertia with a period of $A/(C-A)=305$ days (Euler period). Here $C$ is the earth's polar moment of inertia, and $A = B$ is the mean equatorial moment (rotational symmetry assumed). The difference between the Chandler and the Euler period results from the non-rigidity of the earth. This consideration neglects the fact that there is a small deviation between the axis of rotation and the axis of angular momentum, which is invariable in space. However, the deviation is less than 0.001″ with periods < 1 d.

- The Chandler wobble is superposed by an *annual oscillation* forced by seasonal displacements of air and water masses. It proceeds in the same direction as the Chandler wobble with amplitudes of 0.05″ to 0.1″.
- A *secular motion* of the pole has been observed for more than 100 years. The motion consists of an irregular drift of about 0.003″/year in the direction of the 80° W meridian. Secular motion is mainly due to the melting of the polar ice and to large-scale tectonic movements; it attains large amounts over geological epochs: *polar wander*.
- More *irregular variations* occur at time scales from a few days to years with amplitudes up to 0.02″. They originate primarily from mass redistributions within the atmosphere, but variations due to ocean volume changes, ground water variations, and earthquakes also occur.

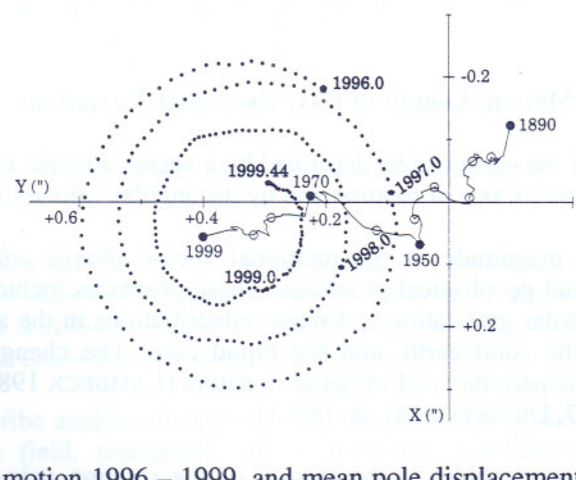


*Fig. 2.9.* Polar motion 1996 – 1999, and mean pole displacement 1890 – 1999, from IERS Annual Report 1998

The superposition of these components results in a slightly perturbed spiral like curve of the instantaneous pole with a slowly advancing mean position (Fig. 2.9). Over one year, the deviations from the mean position remain < 0.3″, corresponding to 9 m on the earth's surface.

The reference for describing the actual position of the pole with respect to the solid earth is provided by the *IERS reference pole*. It agrees within ±0.03″ with the *Conventional International Origin* (CIO), which was defined by the mean position of the north pole as determined between 1900.0 and 1906.0. The position of the instantaneous pole (Celestial Ephemeris Pole, cf. [2.4.2]) with respect to the reference pole is given by the rectangular coordinates $x_P$, $y_P$, which are defined in the plane tangential to the pole. The $x_P$-axis is in the direction of the Greenwich mean meridian (consistent with the previous BIH zero meridian), and the $y_P$-axis is directed along the 90°W meridian. These plane coordinates are usually expressed as spherical distances (in units of arcsec) on the unit sphere.

The *angular velocity* $\omega$ of the earth's rotation, as monitored from the earth, changes with time. Relative changes may reach several $10^{-8}$, which corresponds to several ms for one day. The variations are generally described by the excess revolution time with respect to 86 400 s and then called *Length Of Day* (LOD). They are derived by comparing astronomical time determinations, which deliver Universal Time UT1, with the uniform time scales TAI or UTC, cf. [2.2.2].

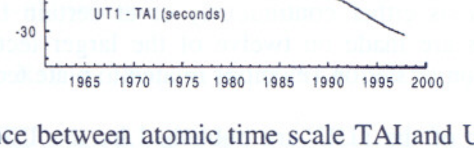


*Fig. 2.10.* Difference between atomic time scale TAI and Universal time UT1 (1962-1998) and length of day (LOD 1979-1987), from IERS Inform. 1998

The following components of LOD variations have been observed (Fig. 2.10):

- A *secular decrease* in the angular velocity of the earth's rotation is caused mainly by tidal friction. It lengthens the day by about 2 ms/century (STEPHENSON and MORRISON 1994).

- Fluctuations over *decades* are due to motions in the earth's liquid core and to slow climatic variations.
- The *tides* of the solid earth and the oceans produce variations of about 1 ms with long (annually) and short (monthly and less) periodic parts.
- *Seasonal* effects are explained by atmospheric excitation, with contributions from water and ice budget variations.
- More *irregular* oscillations stem from different sources, such as terrestrial mass displacements (earthquakes), solar activity, and atmospheric events, e.g., El Niño.

While the effect of polar motion on observations is dependent on location, LOD changes act uniformly on all points. The pole coordinates and LOD, as well as ω, are provided as *Earth Orientation Parameters* (EOP) by the IERS with daily resolution and accuracy of ±0.0003″ resp. ±0.02 ms or better (REIGBER and FEISSEL 1997).

The position of the *geocenter* (origin of the terrestrial reference system) changes slightly in time with respect to the monitoring observatories. Annual and semiannual variations have been found, with amplitudes of several mm/year, from the analysis of satellite orbits. The variations are caused primarily by mass redistributions in the atmosphere and the oceans and by continental water variations. Through the coordinates of the ITRF stations, cf. [2.5.3], the geocenter is given with an accuracy of a few mm (DONG et al. 1997, RAY 1999).

### 2.5.3 International Terrestrial Reference Frame

The International Terrestrial Reference System is realized by the IERS through a global set of space geodetic observing sites. The geocentric Cartesian coordinates and velocities of the observing sites comprise the International Terrestrial Reference Frame (ITRF). The stations participating in the ITRF carry out observations either continuously or at certain time intervals (Fig. 2.11). Observations are made on twelve of the larger tectonic plates, which permits the derivation of station velocities related to plate tectonics, cf. [8.2.3].

Annual realizations of the ITRF are published by the IERS. The ITRF97 is comprised of the geocentric positions $(X,Y,Z)$ for more than 550 stations at about 320 sites and corresponding site velocities (BOUCHER et al. 1999). The accuracy of the results depends on the observation techniques and is maximum for VLBI, SLR, and GPS observations (±0.5...2 cm and ±1...3 mm/year resp.). Several time variable effects are taken into account, including displacements due to the solid earth tides, ocean and atmospheric loading effects, and postglacial rebound, cf. [8.2.2]. The ITRF solutions satisfy the condition of no residual net-rotation relative to the plate tectonics model NNR-NUVEL1A; vertical movements are not allowed at all, cf. [8.2.3]. The orientation of the

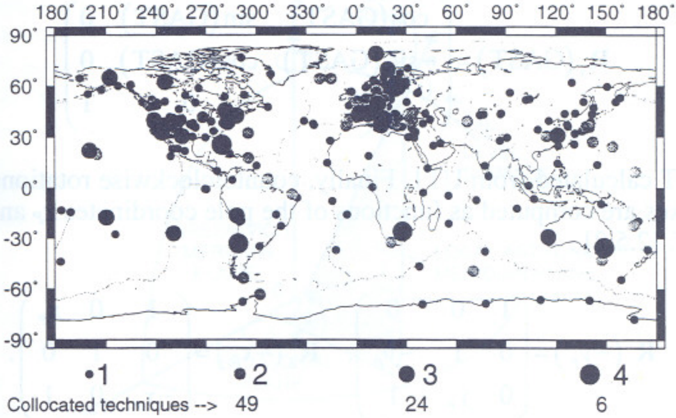


*Fig. 2.11.* International Terrestrial Reference Frame (ITRF) sites 1997, from BOUCHER et al. (1999)

ITRF is given with respect to the IERS reference pole and reference meridian, cf. [2.5.2]. The actual (time t) position vector **r** of a point on the earth's surface is derived from its position at the reference epoch ($t_0$) by

$$\mathbf{r}(t) = \mathbf{r}_0 + \dot{\mathbf{r}}_0 (t - t_0), \tag{2.15}$$

where $\mathbf{r}_0$ and $\dot{\mathbf{r}}_0$ are the position and velocity respectively at $t_0$.

The relation between the celestial (ICRS) and the terrestrial (ITRS) reference systems is given by spatial rotations, which depend on the earth rotation parameters introduced in [2.2.2], [2.4.2], [2.5.2], SEEBER (1993), MCCARTHY (1996), RICHTER, BU. (1995). The complete transformation from the celestial to the terrestrial system reads as

$$\mathbf{r}(\text{ITRS}) = \mathbf{R}_2(-x_P)\mathbf{R}_1(-y_P)\mathbf{R}_3(\text{GAST})\mathbf{N}(t)\mathbf{P}(t)\mathbf{r}(\text{ICRS}). \tag{2.16}$$

The position vector as given in the ICRS is first transformed by the precession matrix **P**($t$) from the reference epoch $t_0$ (J 2000.0) to the observation epoch $t$. The nutation matrix **N**($t$) then transforms from the mean to the instantaneous true equator and vernal equinox. The Eulerian angles in these two rotation matrices are given in the models for precession and nutation, cf. [2.4.2]. The apparent Greenwich sidereal time GAST, cf. [2.2.2], is used to rotate the system about the $Z$-axis:

$$\mathbf{R}_3(\mathrm{GAST}) = \begin{pmatrix} \cos(\mathrm{GAST}) & \sin(\mathrm{GAST}) & 0 \\ -\sin(\mathrm{GAST}) & \cos(\mathrm{GAST}) & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{2.17}$$

with GAST calculated from UT1. Finally, counterclockwise rotations about the *X* and *Y*-axes are computed as functions of the pole coordinates $x_P$ and $y_P$ (small angles), cf. [2.5.2] .

$$\mathbf{R}_1(-y_P) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -y_P \\ 0 & y_P & 1 \end{pmatrix}, \quad \mathbf{R}_2(-x_P) = \begin{pmatrix} 1 & 0 & x_P \\ 0 & 1 & 0 \\ -x_P & 0 & 1 \end{pmatrix}. \tag{2.18}$$

Equations (2.17) and (2.18) provide the transformation from the instantaneous space-fixed system to the conventional terrestrial system.

## 2.6 Gravity Field Related Reference Systems

Most geodetic and astronomic observations on or close to the earth's surface refer to the earth's gravity field by orientation along the local vertical. Consequently, local gravity-field-related reference systems are introduced for the modeling of these observations. The orientation of the local systems with respect to the global reference system is given by astronomic latitude and longitude [2.6.1]. These orientation parameters are used for transformation from the local systems into the global system and back [2.6.2].

### 2.6.1 Orientation of the Local Vertical

The direction of the plumb line (local vertical) with respect to the global geocentric system is given by two angles (Fig. 2.12). The *astronomic (geographic) latitude* $\Phi$ is the angle measured in the plane of the meridian between the equatorial plane and the local vertical through the point *P*. It is reckoned positive from the equator northward and negative to the south. The angle measured in the equatorial plane between the Greenwich meridian plane and the plane of the meridian passing through *P* is the *astronomic (geographic) longitude* $\Lambda$; it is reckoned positive toward the east. The *gravity potential W* locates *P* in the system of level surfaces $W$ = const., cf. [3.2.1]. The *local astronomic meridian plane* is spanned by the local vertical at *P* and a line parallel to the rotational axis, cf. [2.4.1].

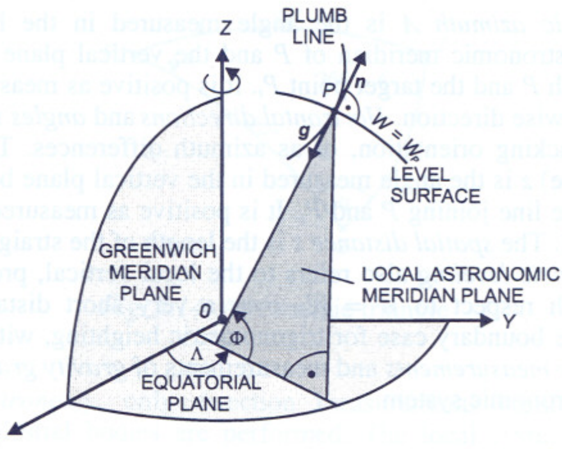


*Fig. 2.12.* Astronomic latitude and longitude

We introduce the outer surface normal **n** (unit vector), which is normal to the level surface $W = W_P$ and passes through $P$. It is directed to the zenith, which is opposite of the direction of the gravity vector **g**. From Fig. 2.12, we see that

$$\mathbf{n} = -\frac{\mathbf{g}}{g} = \begin{pmatrix} \cos\Phi\cos\Lambda \\ \cos\Phi\sin\Lambda \\ \sin\Phi \end{pmatrix}. \tag{2.19}$$

Latitude $\Phi$ and longitude $\Lambda$ can be determined by the methods of geodetic astronomy, cf. [5.3]. Together with the potential $W$, they form a triple of three-dimensional coordinates defined in the gravity field, cf. [3.2.3].

## 2.6.2 Local Astronomic Systems

Geodetic and astronomic observations are tied to the direction of the plumb line at the point of observation and thereby to the earth's gravity field. An exception is distance measurements, which are independent of the reference system. Thus, these observations establish local gravity-field related systems: *Local astronomic systems* (Fig. 2.13). Their origin is at the point of observation $P$. The $z$-axis coincides with the local vertical and points toward the zenith. The $x$-axis (north) and the $y$-axis (east) span the horizontal plane, which is tangent to the level surface $W = W_P$. This $x,y,z$-system is left-handed.

Observable geometric quantities include astronomic azimuths, horizontal directions and angles, zenith angles, spatial distances, and leveled height differences.

The *astronomic azimuth* $A$ is the angle measured in the horizontal plane between the astronomic meridian of $P$ and the vertical plane spanned by the vertical through $P$ and the target point $P_i$. It is positive as measured from the $x$-axis in a clockwise direction. *Horizontal directions* and *angles* may be regarded as azimuths lacking orientation, or as azimuth differences. The *zenith angle* (zenith distance) $z$ is the angle measured in the vertical plane between the local vertical and the line joining $P$ and $P_i$. It is positive as measured from the outer surface normal. The *spatial distance* $s$ is the length of the straight line joining $P$ and $P_i$. *Geometric leveling* also refers to the local vertical, providing a height difference with respect to $W = W_P$ over a very short distance. It may be regarded as the boundary case for trigonometric heighting, with a zenith angle of 90°. *Gravity measurements* and measurements of *gravity gradients* also refer to the local astronomic system.

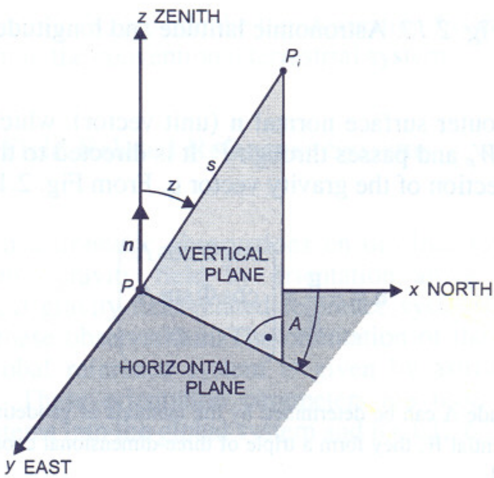


*Fig. 2.13.* Local astronomic system

According to Fig. 2.13, the position vector between $P$ and $P_i$ is given by

$$\mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = s \begin{pmatrix} \cos A \sin z \\ \sin A \sin z \\ \cos z \end{pmatrix}. \tag{2.20}$$

The local astronomic system is used for astronomic and geodetic applications.

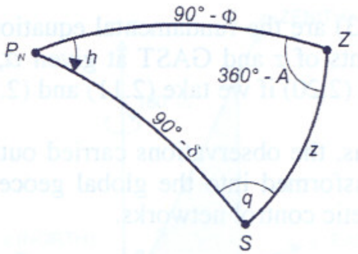


*Fig. 2.14.* Astronomic triangle

In *geodetic astronomy*, only direction measurements (zenith angles and azimuths) to celestial bodies are performed. The local system is called the *horizon system*, and the origin is named topocenter. The points of intersection of the plumb-line direction with the celestial sphere are known as the zenithal point $Z$ and the nadir point $Z'$. The intersection of the horizontal plane with the celestial sphere is the celestial horizon. The azimuth in astronomy is usually reckoned from the south point and is considered positive westward to the north. The relation between the horizon system and the equatorial hour angle system, cf. [2.4.1], is given by the astronomic triangle (Fig. 2.14), see also Fig. 2.4. It is formed on the celestial sphere by the vertices $P_N$ (north pole), $Z$ (zenithal point), and $S$ (celestial body). The triangle contains the compliments to declination ($90° - \delta$) and astronomic latitude ($90° - \Phi$), the hour angle $h$, the zenith angle $z$, the explement of the azimuth ($360° - A$), and the parallactic angle $q$. From spherical astronomy we obtain the transformations

$$\left.\begin{aligned} \cos A \sin z &= \sin\delta\cos\Phi - \cos\delta\cos h \sin\Phi \\ \sin A \sin z &= -\cos\delta \sin h \\ \cos z &= \sin\delta\sin\Phi + \cos\delta\cos h\cos\Phi \end{aligned}\right\}. \tag{2.21}$$

Here the azimuth $A$ is reckoned in the geodetic sense, i.e., positive from the north.

The transition to the $\alpha,\delta$-system ($\alpha$ = right ascension) is given by the local apparent sidereal time LAST, see (2.12):

$$\alpha = \text{LAST} - h\,. \tag{2.22}$$

Astronomic longitude $\Lambda$ is obtained by comparing LAST with the Greenwich sidereal time (2.5):

$$\Lambda = \text{LAST - GAST}\,. \tag{2.23}$$

Equations (2.21) to (2.23) are the fundamental equations for determining Φ, Λ and $A$ from measurements of $z$ and GAST at given $\alpha$, $\delta$, cf. [5.3.2]. Equation (2.21) also follows from (2.20) if we take (2.11) and (2.29) into account.

For *geodetic* applications, the observations carried out in the local astronomic systems have to be transformed into the global geocentric system for further use in establishing geodetic control networks.

Due to the non-parallelism of the plumb lines, the orientation of the local systems depends on position and thus changes rapidly from place to place. Computations in *one* individual system are therefore admissible only in very limited areas when applying formulas of plane geometry.

The plumb line direction can be referred to the global geocentric-system by means of the "orientation" parameters astronomic latitude Φ and longitude Λ (Fig. 2.15). After a parallel shift of the global system into the local one (Fig. 2.16), we transform the latter one to a right-handed system by applying the reflection matrix

$$\mathbf{S}_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{2.24}$$

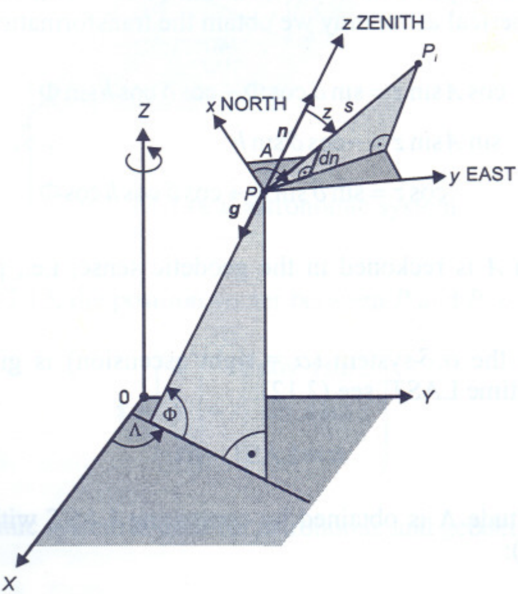


*Fig. 2.15.* Local astronomic and global geocentric system

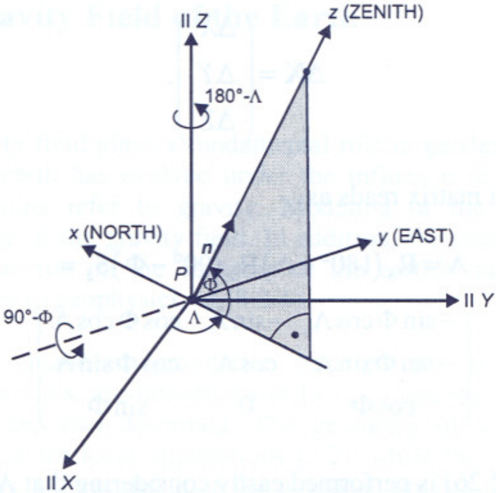


*Fig. 2.16* Transformation between the local astronomic and the geocentric system

We then rotate the local system by $90° - \Phi$ around the (new) $y$-axis and by $180° - \Lambda$ around the $z$-axis with the rotation matrices

$$\mathbf{R}_2\left(90° - \Phi\right) = \begin{pmatrix} \sin\Phi & 0 & -\cos\Phi \\ 0 & 1 & 0 \\ \cos\Phi & 0 & \sin\Phi \end{pmatrix} \quad \text{and}$$

$$\mathbf{R}_3\left(180° - \Lambda\right) = \begin{pmatrix} -\cos\Lambda & \sin\Lambda & 0 \\ -\sin\Lambda & -\cos\Lambda & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{2.25}$$

Coordinates differences between $P_i$ and $P$ in the geocentric system are thus obtained by

$$\Delta\mathbf{X} = \mathbf{A}\mathbf{x}\,, \tag{2.26}$$

with $\mathbf{x}$ given by (2.20) and

$$\Delta \mathbf{X} = \begin{pmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{pmatrix}. \tag{2.27}$$

The transformation matrix reads as

$$\mathbf{A} = \mathbf{R}_3(180° - \Lambda)\mathbf{R}_2(90° - \Phi)\mathbf{S}_2 = \begin{pmatrix} -\sin\Phi\cos\Lambda & -\sin\Lambda & \cos\Phi\cos\Lambda \\ -\sin\Phi\sin\Lambda & \cos\Lambda & \cos\Phi\sin\Lambda \\ \cos\Phi & 0 & \sin\Phi \end{pmatrix}. \tag{2.28}$$

The inversion of (2.26) is performed easily considering that **A** is orthonormal:

$$\mathbf{A}^{-1} = \mathbf{A}^{\mathrm{T}}.$$

We obtain

$$\mathbf{x} = \mathbf{A}^{-1}\Delta\mathbf{X}, \tag{2.29}$$

with

$$\mathbf{A}^{-1} = \begin{pmatrix} -\sin\Phi\cos\Lambda & -\sin\Phi\sin\Lambda & \cos\Phi \\ -\sin\Lambda & \cos\Lambda & 0 \\ \cos\Phi\cos\Lambda & \cos\Phi\sin\Lambda & \sin\Phi \end{pmatrix}. \tag{2.30}$$

Equations (2.27) to (2.30) are the basic equations for the evaluation of local geodetic measurements within the three-dimensional reference frame, cf. [6.2.1].