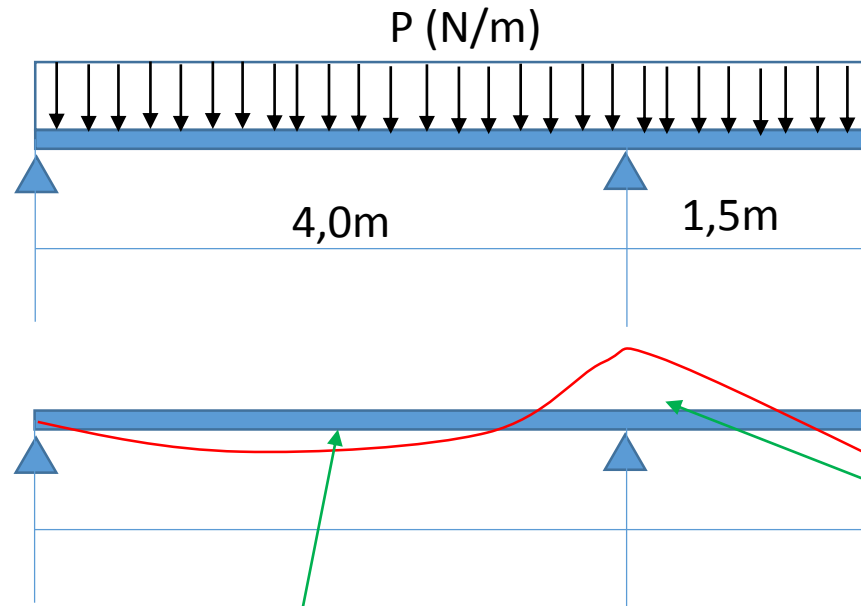


Exercício de flexão simples

Prof. Dr. Carlos Aurélio Nadal

Seja a viga abaixo representada pelo seu diagrama simplificado onde $P=30\text{kN/m}$, $E=205\text{GPa}$, calcular a tensão máxima de compressão e a tensão máxima de tração na viga.

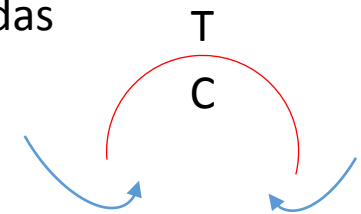
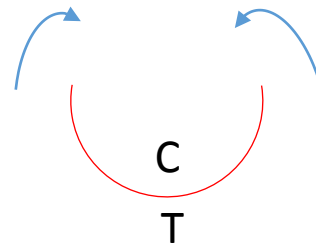


Linha elástica

Flexão positiva $s=$ comprimidas
 $i=$ tracionadas

Flexão negativa $i=$ comprimidas
 $s=$ tracionadas

$s=$ fibras superiores
 $i=$ fibras inferiores



Secção transversal

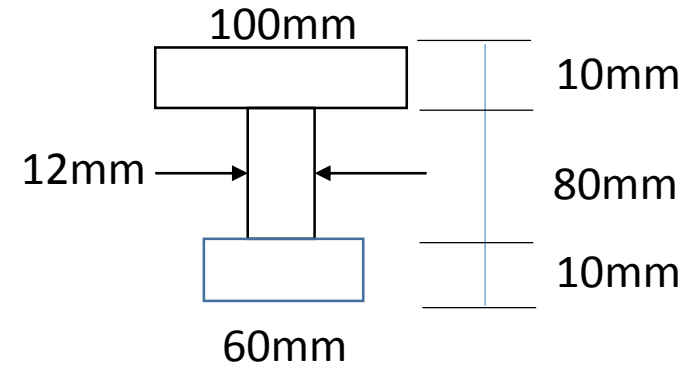
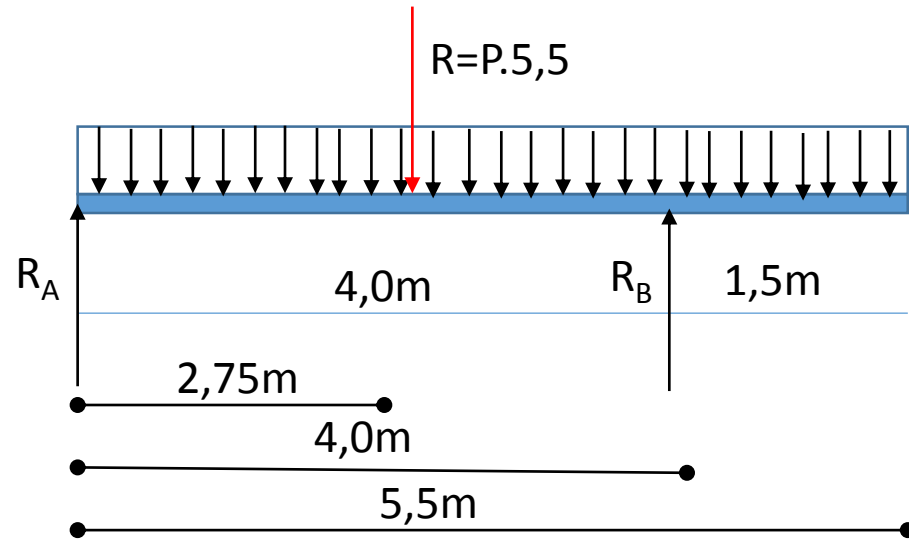
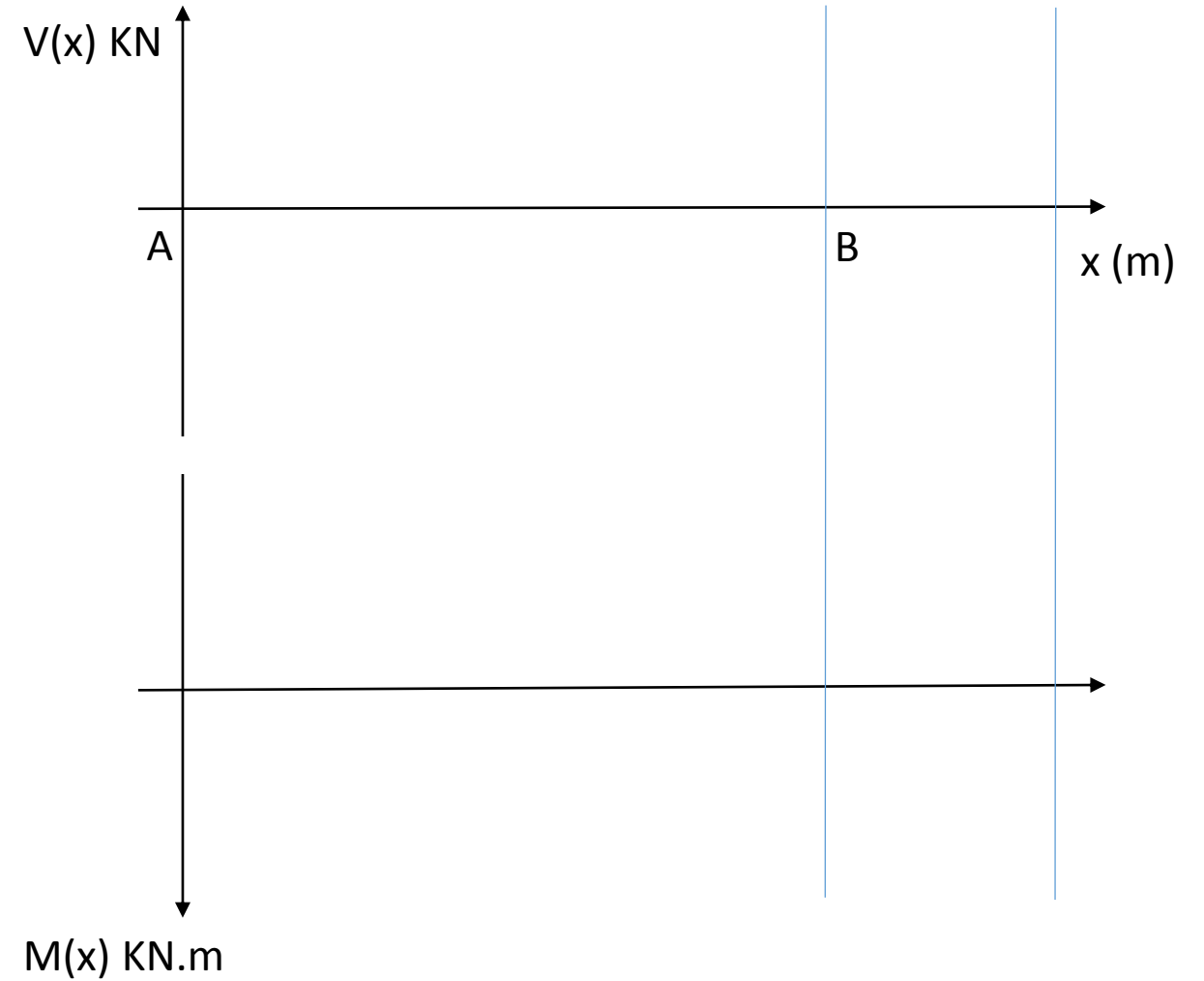


Diagrama de corpo livre da viga

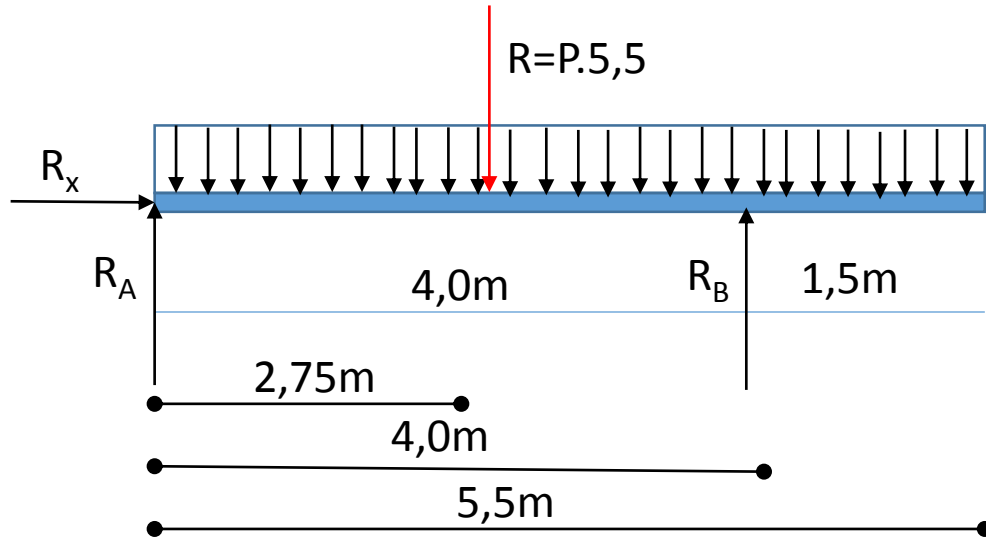


Força cortante



Cálculo das reações

Condição de equilíbrio: $\Sigma M=0$; $\Sigma F_x=0$ e $\Sigma F_y=0$



$$\Sigma M_A=0$$

$$-5,5 \times P \times 2,75 + R_B \times 4 = 0$$

$$R_B = 3,78P$$

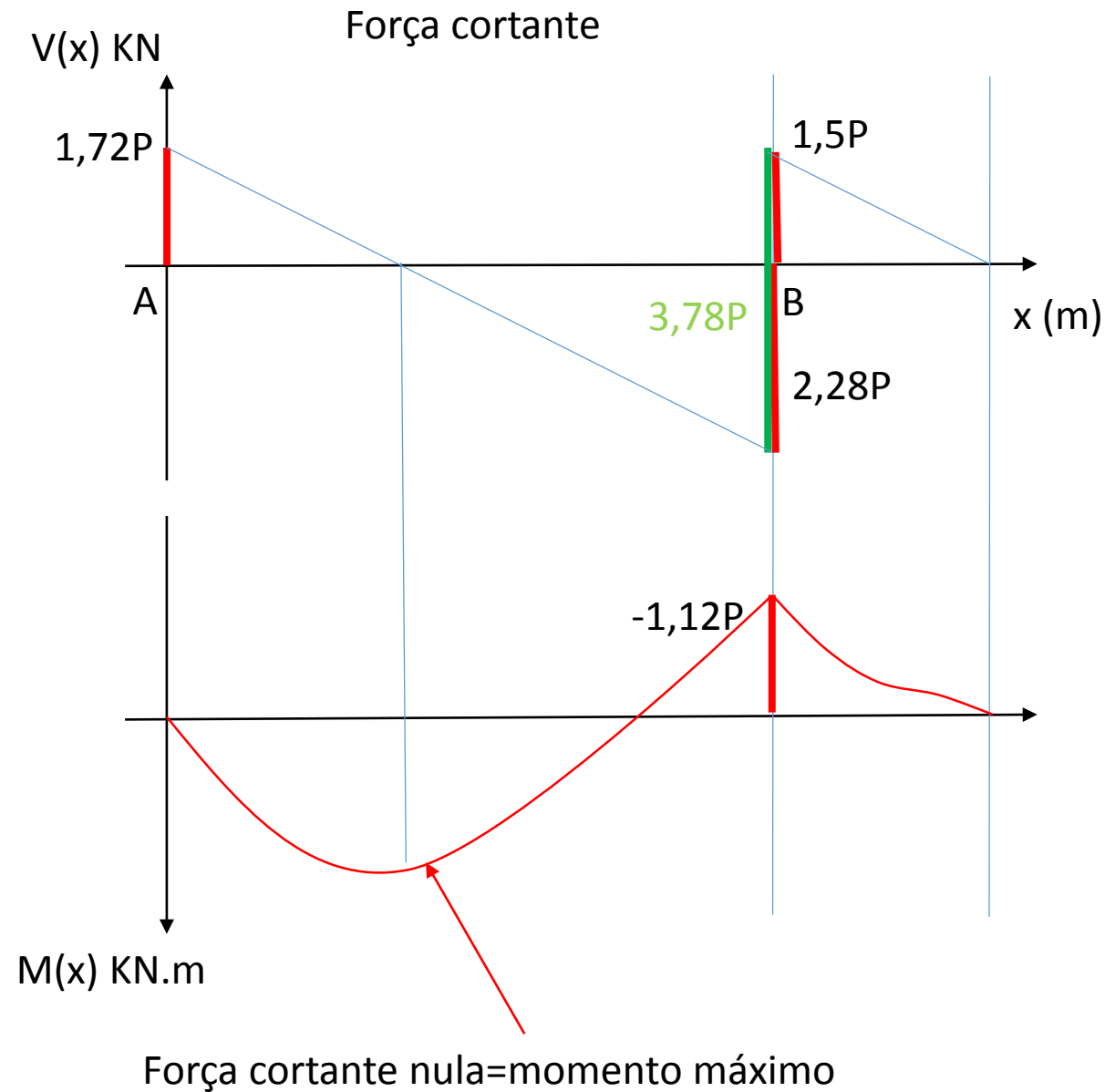
$$\Sigma F_x=0$$

$R_x=0$ Forças horizontais

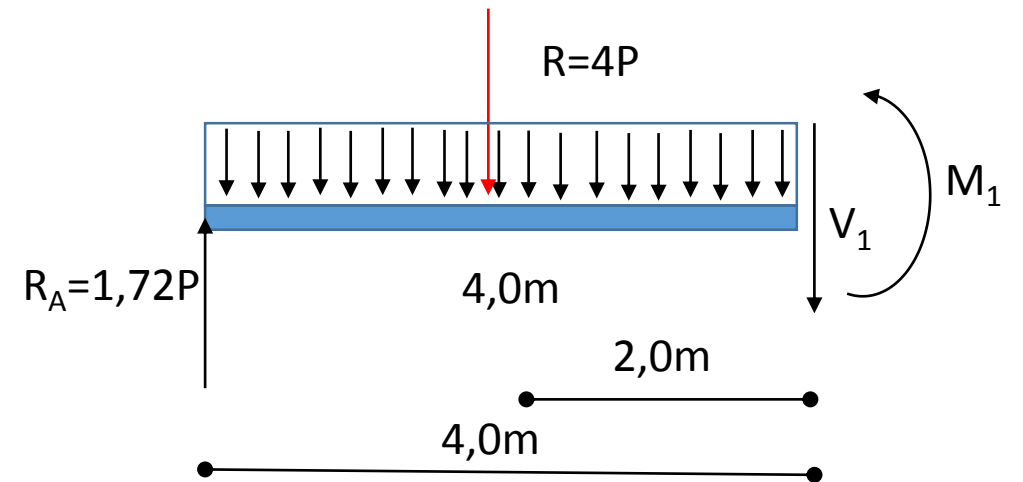
$$\Sigma F_y=0$$

$$R_A - R + R_B = 0$$

$$R_A - 5,5P + 3,78P = 0 \quad \therefore R_A = 1,72P$$



Análise da seção 1 que passa pelo ponto B para o desenho da força cortante.



$$\Sigma M_1 = 0$$

$$M_1 + 4P \times 2 - 1,72P \times 4 = 0$$

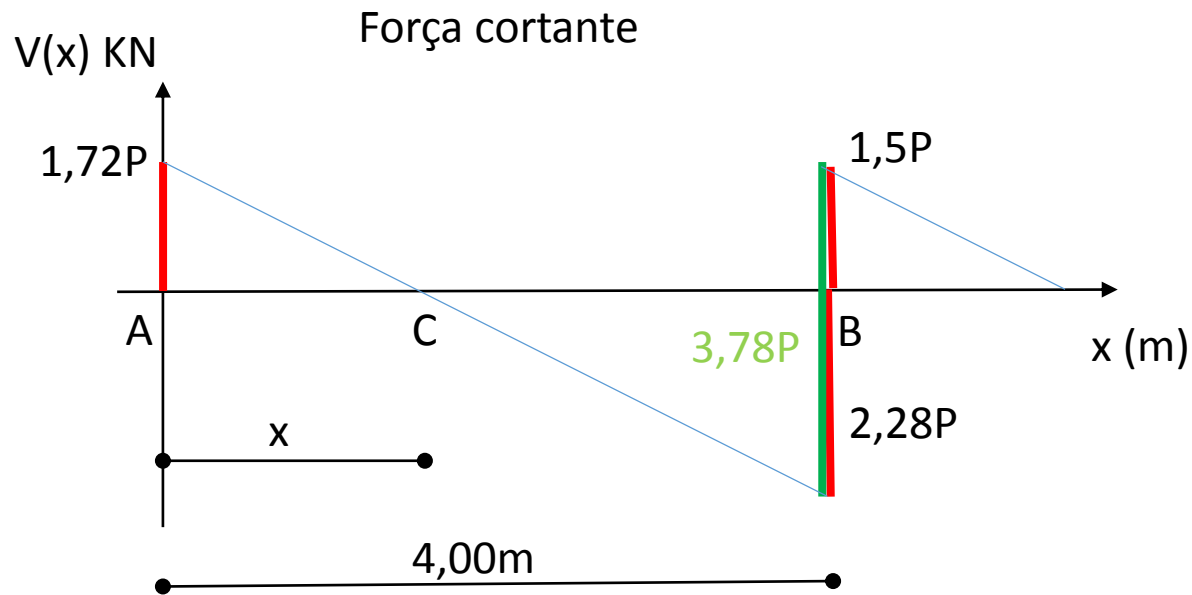
$$M_1 = -1,12P$$

$$\Sigma F_y = 0$$

$$1,72P - 4P - V_1 = 0$$

$$V_1 = -2,28P$$

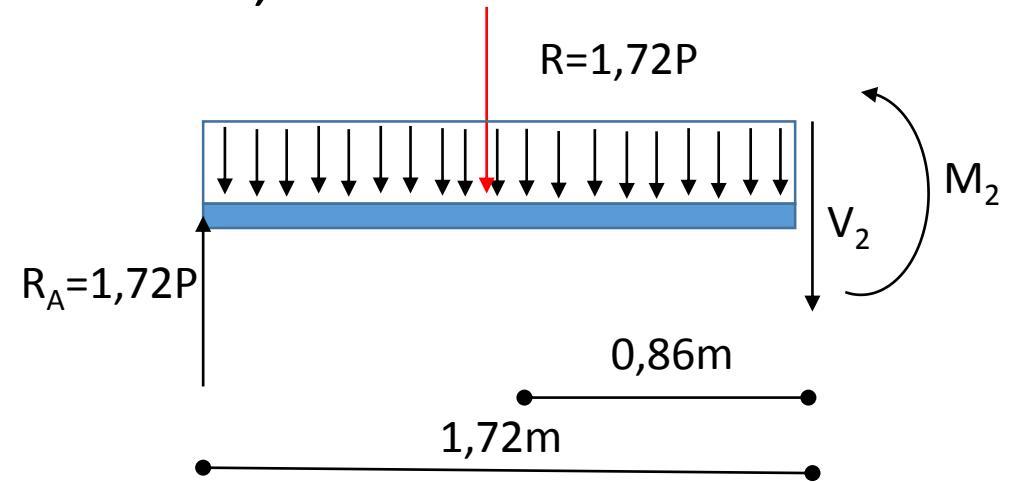
$$V_1 - R_B = -2,28P + 3,78P = 1,5P$$



$$x = 1,72P$$

$$4 = 1,72P + 2,28P = 4P$$

$$x = 1,72m$$

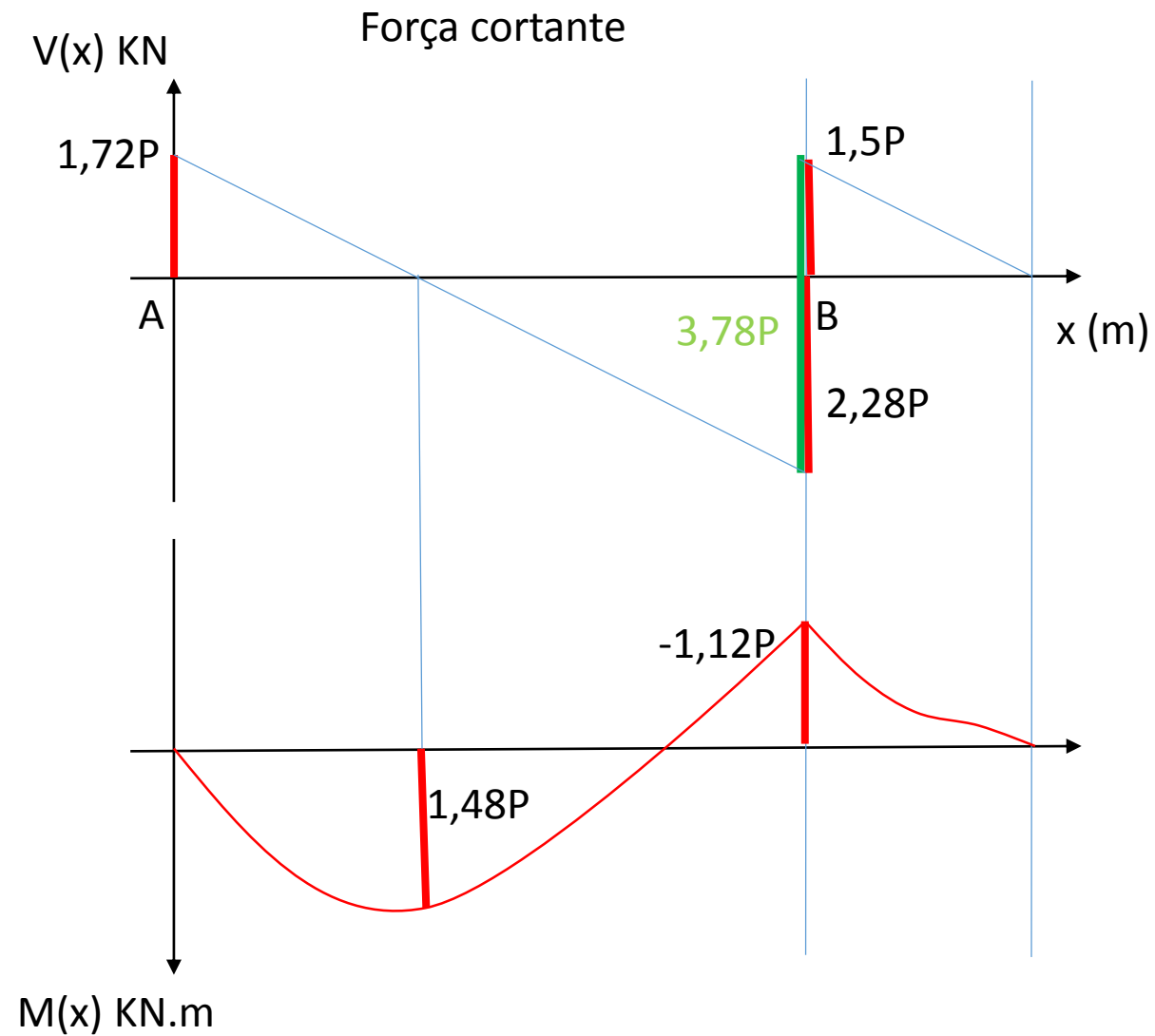


Cálculo na seção C

$$\Sigma M_2 = 0$$

$$M_2 + 1,72P \times 0,86 - 1,72P \times 1,72 = 0$$

$$M_2 = 1,48P$$



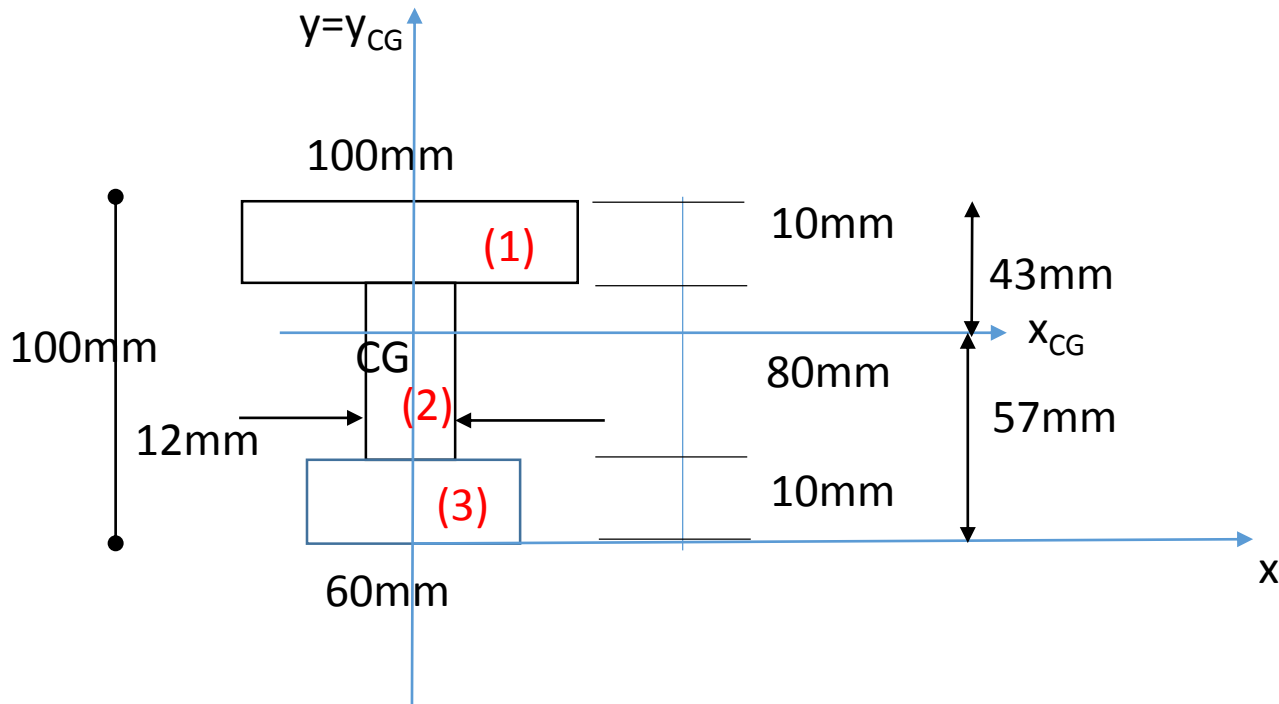
Momento máximo

$$M_{\max} = 1,48P$$

Momento mínimo

$$M_{\min} = -1,12P$$

Cálculo do momento de inércia da seção transversal



Determinação do centro de gravidade da seção

$$Y_1=95\text{mm} \quad A_1=10 \times 100=1000\text{mm}^2$$

$$Y_2=50\text{mm} \quad A_2=12 \times 80=960\text{mm}^2$$

$$Y_3=5\text{mm} \quad A_3=60 \times 10=600\text{mm}^2$$

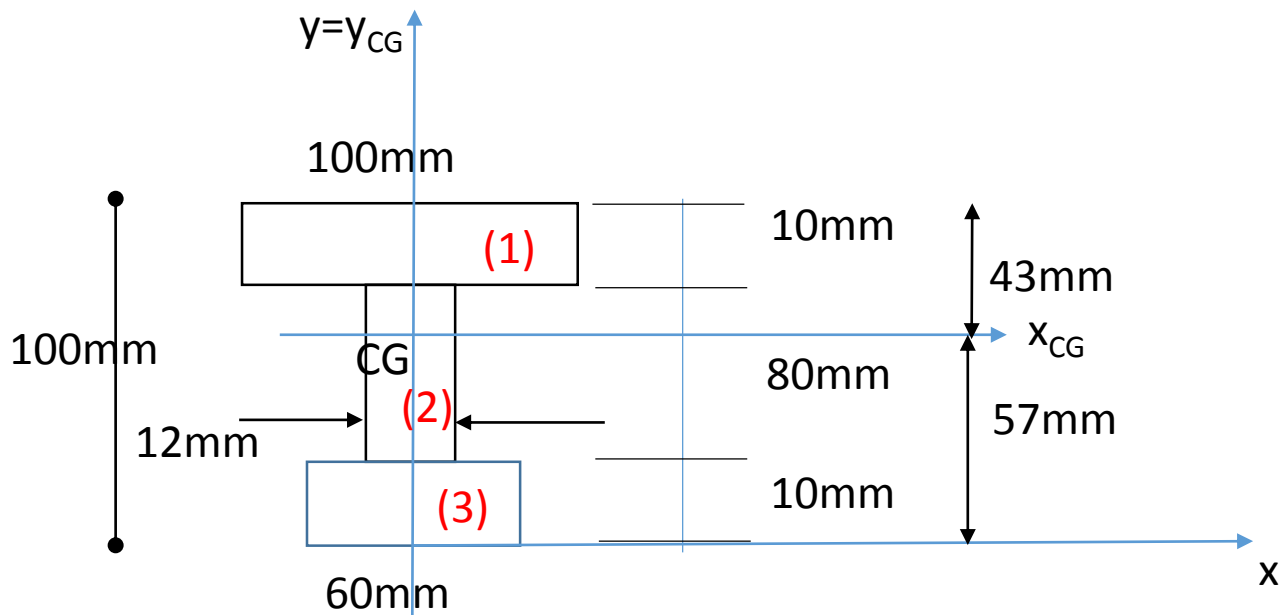
$$Y_{CG}=\frac{\sum A_i y_i}{\sum A_i}$$

$$Y_{CG}=\frac{(1000 \times 95 + 960 \times 50 + 600 \times 5)}{(1000 + 960 + 600)}$$

$$Y_{CG}=57\text{mm}$$

Momento de Inércia em relação ao eixo principal de inércia x_{CG}

$$I_{x_{CG}} = I_{1x_{CG}} + I_{2x_{CG}} + I_{3x_{CG}}$$



$$I_{2x_{CG}} = bh^3/12 + A_2xd_2^2$$

$$I_{2x_{CG}} = 0,012 \times 0,08^3 / 12 + 0,012 \times 0,08 \times 0,007^2$$

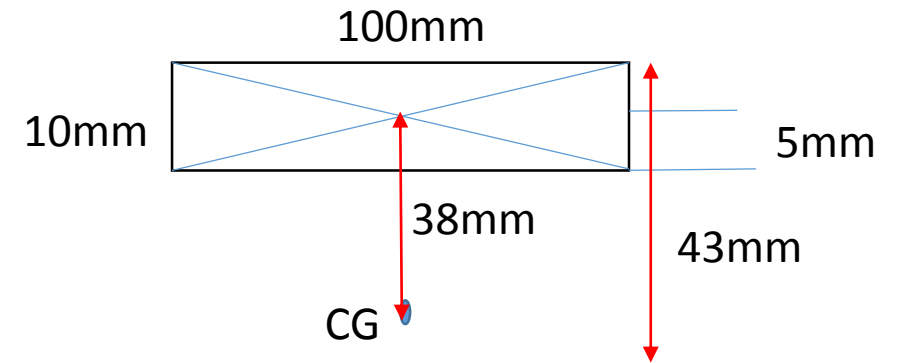
$$I_{2x_{CG}} = 5,59 \times 10^{-7} \text{ m}^4$$

$$I_{3x_{CG}} = bh^3/12 + A_2xd_2^2$$

$$I_{3x_{CG}} = 0,06 \times 0,01^3 / 12 + 0,06 \times 0,01 \times 0,052^2$$

$$I_{3x_{CG}} = 1,63 \times 10^{-6} \text{ m}^4$$

Para figura 1



$$I_{1x_{CG}} = bh^3/12 + A_1xd_1^2$$

$$I_{1x_{CG}} = 0,1 \times 0,01^3 / 12 + 0,1 \times 0,01 \times 0,038^2$$

$$I_{1x_{CG}} = 1,452 \times 10^{-6} \text{ m}^4$$

$$I_{x_{CG}} = 3,641 \times 10^{-6} \text{ m}^4$$

$$M_{\max} = 1,48P$$

$$M_{\min} = -1,12P$$

$$I_{x_{CG}} = 3,641 \times 10^{-6} \text{ m}^4$$

Para o momento máximo tem-se

$$\sigma = My/I$$

Tensão de tração

$$\sigma_T = 44,4 \times 10^3 \times 0,057 / 3,641 \times 10^{-6} \text{ (divide o resultado por 1000000 para Mpa)}$$

$$\sigma_T = 695 \text{ MPa}$$

Tensão de compressão

$$\sigma_C = 44,4 \times 10^3 \times 0,043 / 3,641 \times 10^{-6}$$

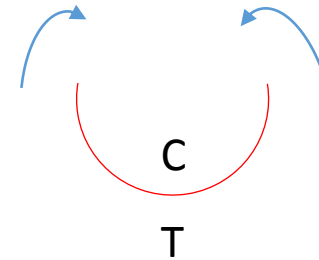
$$\sigma_C = 524,4 \text{ MPa}$$

$$M_{\max} = 44,4 \text{ kNm}$$

$$M_{\min} = 33,6 \text{ kNm}$$

$$x = 1,72 \text{ m}$$

$$x = 4 \text{ m}$$



$$M_{\max} = 1,48P$$

$$M_{\min} = -1,12P$$

$$I_{x_{CG}} = 3,641 \times 10^{-6} \text{ m}^4$$

Para o momento mínimo ou o máximo negativo tem-se

$$\sigma = My/I$$

Tensão de tração

$$\sigma_T = 33,6 \times 10^3 \times 0,043 / 3,641 \times 10^{-6} \text{ (divide o resultado por 1000000 para Mpa)}$$

$$\sigma_T = 396,8 \text{ MPa}$$

Tensão de compressão

$$\sigma_C = 33,6 \times 10^3 \times 0,057 / 3,641 \times 10^{-6}$$

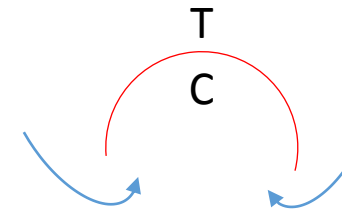
$$\sigma_C = 526 \text{ MPa}$$

$$M_{\max} = 44,4 \text{ kNm}$$

$$M_{\min} = 33,6 \text{ kNm}$$

$$x = 1,72 \text{ m}$$

$$x = 4 \text{ m}$$



Tensão máxima de tração

$$\sigma_{\text{máxT}} = 695 \text{MPa}$$

Tensão máxima de compressão

$$\sigma_{\text{máxC}} = 526 \text{MPa}$$